Teaching and Learning Center
University of Washington Tacoma

Completing the Square

The method of completing the square is sometimes used to convert a quadratic function in this form ***f(x)=ax²+bx+c*** into vertex form ***f(x)=a(x-h)²+k*** where the vertex is (h,k).

Start with a quadratic function in ax^2+bx+c form.

$$f(x) = ax^2 + bx + c$$

Separate the first two terms with parentheses.

$$f(x) = (ax^2 + bx) + c$$

Factor out the constant a from each term in the parentheses. Leave room after the $\frac{b}{a}x$ term.

$$f(x) = a\left(x^2 + \frac{b}{a}x \qquad \right) + c$$

Take the coefficient in front of the x term and divide it by 2 then square it. $\frac{b}{a} \rightarrow \left(\frac{b}{2a}\right)^2$. Add that value inside the parentheses.

$$f(x) = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c$$

The value $\left(\frac{b}{2a}\right)^2$ was added inside the parentheses and everything inside the parentheses is being multiplied by a so you have actually added $a\left(\frac{b}{2a}\right)^2$ to the entire function. For everything to remain equal we must subtract $a\left(\frac{b}{2a}\right)^2$ from the entire function. Subtract $a\left(\frac{b}{2a}\right)^2$ outside the parentheses.

$$f(x) = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b}{2a}\right)^2$$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$ is now a perfect square so it can be condensed into $\left(x + \frac{b}{2a}\right)^2$.

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + c - a\left(\frac{b}{2a}\right)^2$$

f(x) is now in vertex form with the vertex (*h,k*) being $\left(-\frac{b}{2a},\ c - a\left(\frac{b}{2a}\right)^2\right)$. You have now completed the square.

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Examples

Start with $f(x) = x^2 + 2x - 17$.

$$f(x) = x^2 + 2x - 17$$

Separate the first two terms with parentheses.

$$f(x) = (x^2 + 2x \quad) - 17$$

There is no need to factor out the coefficient in front of the x^2 term because it is already 1. Divide the coefficient in front of the x term by 2 then square it. $2 \rightarrow \left(\frac{2}{2}\right)^2 = 1$. Add 1 inside the parentheses.

$$f(x) = (x^2 + 2x + 1) - 17$$

Since you added 1 to the function, you must subtract 1 from the function to keep everything equal. Subtract 1 outside the parentheses.

$$f(x) = (x^2 + 2x + 1) - 17 - 1$$

$x^2 + 2x + 1$ is a perfect square so it can be condensed into $(x+1)^2$.

$$f(x) = (x+1)^2 - 18$$

$f(x)$ is now in vertex form. The vertex is (-1, -18).

Start with $f(x) = 5x^2 - 3x + 20$.

$$f(x) = 5x^2 - 3x + 20$$

Separate the first two terms with parentheses.

$$f(x) = (5x^2 - 3x \quad) + 20$$

Factor 5 out of the parentheses.

$$f(x) = 5\left(x^2 - \frac{3}{5}x \quad \right) + 20$$

Divide $-\frac{3}{5}$ by 2 then square it. $-\frac{3}{5} \rightarrow \left(-\frac{3}{10}\right)^2 = \frac{9}{100}$. Add $\frac{9}{100}$ inside the parentheses.

$$f(x) = 5\left(x^2 - \frac{3}{5}x + \frac{9}{100}\right) + 20$$

The $\frac{9}{100}$ is being multiplied by 5 so you must subtract $5\left(\frac{9}{100}\right)$ from the function to keep everything equal. $5\left(\frac{9}{100}\right) = \frac{9}{20}$. Subtract $\frac{9}{20}$ outside the parentheses.

$$f(x) = 5\left(x^2 - \frac{3}{5}x + \frac{9}{100}\right) + 20 - \frac{9}{20}$$

$x^2 - \frac{3}{5}x + \frac{9}{100}$ is a perfect square which can be condensed into $\left(x - \frac{3}{10}\right)^2$.

$$f(x) = 5\left(x - \frac{3}{10}\right)^2 + \frac{391}{20}$$

$f(x)$ is now in vertex form. The vertex is $\left(\frac{3}{10}, \frac{391}{20}\right)$.